A two-level optimization procedure for material characterization of composites using two symmetric angle-ply beams

C.M. Chen, T.Y. Kam*

Mechanical Engineering Department, National Chiao Tung University, Hsin Chu, 300 Taiwan, ROC

Abstract

A two-level optimization procedure for determining elastic constants E_1, E_2, G_{12}, and v_{12} of laminated composite materials using measured axial and lateral strains of two symmetric angle-ply beams with different fiber angles subjected to three-point-bending testing is presented. In the first-level optimization process, the theoretically and experimentally predicted axial and lateral strains of a $[(45°/-45°)_6]_s$ beam are used to construct the strain discrepancy function which is a measure of the sum of the squared differences between the experimental and theoretical predictions of the axial and lateral strains. The identification of the material constants is then formulated as a constrained minimization problem in which the best estimates of shear modulus and Poisson's ratio of the beam are determined to make the strain discrepancy function a global minimum. In the second-level optimization process, shear modulus and Poisson's ratio determined in the first level of optimization are kept constant and Young's moduli of the second angle-ply beam with fiber angles different from 45° are identified by minimizing the strain discrepancy function established at this level of optimization. The suitability of the proposed procedure for material characterization of composite materials has been demonstrated by means of a number of examples.
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1. Introduction

Material constants determination is an important process in the design or quality assurance of laminated composite parts. The attainment of actual material constants can help predict realistic mechanical behaviors or perform reasonable integrity assessments of the laminated composite parts. In determining the material constants, three laminated composite specimens with different fiber angles subjected to static tensile testing are generally used to determine elastic constants E_1, E_2, G_{12}, and v_{12} of the laminated composite specimens. According to the ASTM standards of D3039/3039M and D3518/3518M, two specimens with 0° and 90° fiber angles are used to determine E_1, v_{12}, and E_2 while a 45° specimen is required to determine G_{12}. The preparation of laminated composite specimens and testing of the specimens are

usually time consuming and tedious. To shorten the time for material preparation and simplify the material testing process, it has thus long been desired to have a simple yet effective procedure for elastic constants determination of composite materials. In recent years, many researchers have proposed different techniques to identify the structural or material properties of structures using various measured structural response data. For instance, Berman and Nagy [1] developed a method that used measured normal modes and natural frequencies to improve an analytical mass and stiffness matrix model of a structure. Kam and his associates [2,3] developed methods to identify the element bending stiffnesses of beam structures using measured natural frequencies and mode shapes or displacements alone. A number of researchers developed different combined numerical/experimental methods in which 10–16 experimental eigen frequencies were used to identify elastic properties of laminated composites [4–9]. Wang and Kam [10] proposed a constrained minimization method to identify five material constants of shear-deformable laminated composite plates using measured

*Corresponding author. Tel.: +886 3 5712121x55124;
fax: +886 3 5753735.
E-mail address:* tykam@mail.nctu.edu.tw (T.Y. Kam).

Nomenclature

d_{ij} $(i,j = x,y,s)$ bending compliance coefficients
D_{ij} bending stiffness coefficients
e error function
E_1, E_2 Young's moduli in fiber and matrix directions
F applied load
G_{12} shear modulus in the 1–2 plane
h beam thickness
L beam length
M_x, M_y, M_z moment resultant
$\overline{Q}_{ij}^{(m)}(i,j = x,y,s)$ the transformed reduced stiffness of the mth layer
w beam width
$\underline{x}$ design variables

$\underline{\tilde{x}}$ modified design variables

Greek letters

α_i normalization factors
$\varepsilon_x, \varepsilon_y, \gamma_{xy}$ axial, lateral, and shear strains
$\kappa_x, \kappa_y, \kappa_{xy}$ curvatures in the x-direction, y-direction, and x–y plane
μ_j, η_j, r_p multipliers
$\mu_j^0, \eta_j^0, r_p^0, \gamma_0, r_p^{max}$ parameters
v_{ij} Poisson's ratio
ξ amplification factor

strains and/or displacements. Shin and Pande [11] developed a neural network method to identify the elastic constants of orthotropic materials. Marin et al. [12] used the boundary element method and the strain/displacement measurements on the boundary to identify the elastic constants of isotropic materials.

In the previous study [13], the authors have used three measured strains of a symmetric angle-ply beam subjected to three-point bending to identify elastic constants E_1, E_2, G_{12}, and v_{12} of the composite beam. Although the previously proposed method is simple and easy to use, the percentage error of the identified E_1 may exceed 6%. In this paper, an improved procedure established on the basis of a two-level optimization method is presented for material characterization of composites using two symmetric angle-ply beams with different fiber angles subjected to three-point bending. In the proposed procedure, experimental axial and lateral strains of the symmetric angle-ply beams are used to construct two different strain discrepancy functions which are measures of the sums of the squared differences between the experimental and theoretical predictions of the axial and lateral strains of the beams for, respectively, the two levels of optimization, and a multi-start global minimization technique is used to identify material constants G_{12} and v_{12} at the first-level optimization and material constants E_1 and E_2 at the second-level optimization by minimizing the strain discrepancy functions. The accuracy and feasibility of the proposed procedure are demonstrated by means of several examples on the material constants identification of symmetric angle-ply beams made of different composite materials.

2. Strain analysis of symmetric angle-ply beam

Consider a symmetric angle-ply beam of size $L \times h \times w$ subjected to three-point bending. Let the neutral axis of the beam coincide with the x-axis. The bending moment resultants at the mid-span of the beam determined from

the equilibrium conditions of the beam are

$$M_x = \frac{FL}{4w}, \quad M_y = 0 \quad \text{and} \quad M_z = 0, \tag{1}$$

where M_x, M_y, and M_z are moment resultants; F is the applied line load acting across the width at the mid-span of the beam; w is beam width; L is beam length. Based on the narrow beam theory [14], the relations between the moment resultants and curvatures for the composite beam can be expressed as

$$\begin{Bmatrix} M_x \\ 0 \\ 0 \end{Bmatrix} = \begin{bmatrix} D_{xx} & D_{xy} & D_{xs} \\ D_{yx} & D_{yy} & D_{ys} \\ D_{sx} & D_{sy} & D_{ss} \end{bmatrix} \begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix}, \tag{2}$$

where κ_x, κ_y, and κ_{xy} are curvatures in the x-direction, y-direction, and x–y plane, respectively. The bending stiffness coefficients D_{ij} are expressed as

$$D_{ij} = \int_{-h/2}^{h/2} \overline{Q}_{ij}^{(m)} z^2 \, dz \quad (i,j = x,y,s), \tag{3}$$

where h is the thickness of the composite beam; $\overline{Q}_{ij}^{(m)}(i,j = x,y,s)$ are the transformed reduced stiffnesses of the mth layer with an arbitrary fiber angle; z is the axis in the thickness direction. For an orthotropic lamina, the untransformed reduced stiffnesses are expressed as

$$\underline{Q} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \tag{4}$$

with

$$Q_{11} = \frac{E_1}{1 - v_{12}v_{21}}, \quad Q_{22} = \frac{E_2}{1 - v_{12}v_{21}},$$

$$Q_{12} = Q_{21} = \frac{v_{21}E_1}{1 - v_{12}v_{21}} = \frac{v_{12}E_2}{1 - v_{12}v_{21}},$$

$$Q_{66} = G_{12}, \tag{5}$$

where E_1, E_2 are Young's moduli in fiber and transverse directions, respectively; v_{ij} is Poisson's ratio for transverse

strain in the j-direction when stressed in the i-direction; G_{12} is shear modulus in the 1–2 plane. The relations between $\overline{Q}_{ij}$ and Q_{ij} can be found in the literature [14]. The inversion of Eq. (2) gives

$$\begin{Bmatrix} \kappa_x \\ \kappa_y \\ \kappa_{xy} \end{Bmatrix} = \begin{bmatrix} d_{xx} & d_{xy} & d_{xs} \\ d_{yx} & d_{yy} & d_{ys} \\ d_{sx} & d_{sy} & d_{ss} \end{bmatrix} \begin{Bmatrix} M_x \\ 0 \\ 0 \end{Bmatrix}, \tag{6}$$

where d_{ij} $(i,j = x, y, s)$ are bending compliance coefficients. In view of Eqs. (1) and (6), the strains on the bottom surface at the mid-span of the beam obtained from the strain–curvature relations are

$$\varepsilon_x = \frac{FLhd_{xx}}{8w}, \tag{7a}$$

$$\varepsilon_y = \frac{FLhd_{xy}}{8w}, \tag{7b}$$

$$\gamma_{xy} = \frac{FLhd_{sx}}{8w}, \tag{7c}$$

where ε_x, ε_y, and γ_{xy} are the axial, lateral, and shear strains, respectively.

It is noted that the strains can be determined directly from the above equations when the material constants of the composite beam are available. In an inverse problem, the material constants, however, are unknown and need to be identified from a given set of strains. Since in practice it is much easier and more accurate to measure normal strains ε_x and ε_y than shear strain γ_{xy}, it is thus attempted to determine the material constants from Eqs. (7a) and (7b) using two pairs of measured axial and lateral strains ($\varepsilon_x, \varepsilon_y$) obtained from two symmetric angle-ply beams with different fiber angles. Herein, a two-level optimization procedure is presented to tackle this problem of material constants identification.

It is worth pointing out here that the symmetric angle-ply beam in Fig. 1 when subjected to three-point bending will have lift-off type displacements at the two supporting ends of the beam. Though the narrow beam theory does not take the end lift-offs into consideration, detailed finite element analyses with the consideration of end lift-offs for the symmetric angle-ply beams with

different fiber angles using SHELL 99 element of the commercial code ANSYS have shown that the strains obtained from Eqs. (7) with errors less than 3% are found to be acceptable for the beams with widths smaller than or equal to 12 mm.

3. Identification of material constants

The problem of material constants identification of symmetric angle-ply beams is formulated as a minimization problem. Two pairs of measured axial and lateral strains obtained from two symmetric angle-ply beams with different fiber angles are used to identify four elastic constants of the composite beams in the following minimization problem:

$$\text{minimize} \quad e(\underline{x}) = \sum_{i=1}^{2} [(\varepsilon_{xi}^* - \varepsilon_{xi})^2 + (\varepsilon_{yi}^* - \varepsilon_{yi})^2] \tag{8}$$

$$\text{subject to} \quad \underline{x}^L \leqslant \underline{x} \leqslant \underline{x}^U,$$

where $e(x)$ is a strain discrepancy function measuring the sum of the squared differences between the predicted and measured strains; $\underline{x} = [E_1, E_2, G_{12}, v_{12}]$, the vector containing material constants; ε_{xi}^*, ε_{yi}^* are, respectively, the measured axial and lateral strains of the ith composite beam; ε_{xi}, ε_{yi} are, respectively, the predicted axial and lateral strains of the ith composite beam; $\underline{x}^L$, $\underline{x}^U$ are, respectively, the lower and upper bounds of the elastic constants. It is noted that the direct solution of the above one-level optimization problem using any of the conventional optimization techniques may encounter great difficulty in producing acceptable results. Furthermore, as will be shown, the identified material constants obtained from the direct solution of the above one-level optimization problem are very sensitive to the variations of the strains and thus may become erroneous even when the variations of the measured strains are small. Since the existence of noise-induced variations in measured strains is inevitable, the determination of accurate material constants directly from the above material constants identification problem may become impossible. To tackle this difficulty, herein a two-level optimization procedure is proposed to solve the minimization problem of Eq. (8). In the proposed procedure, the first-level optimization problem is expressed as

$$\text{minimize} \quad e(\underline{x}_1) = [(\varepsilon_{x1}^* - \varepsilon_{x1})^2 + (\varepsilon_{y1}^* - \varepsilon_{y1})^2] \cdot \xi \tag{9}$$

$$\text{subject to} \quad x_{1i}^L \leqslant x_{1i} \leqslant x_{1i}^U; \quad i = 1-4,$$

where $e(\underline{x}_1)$ is the first-level strain discrepancy function measuring the sum of the squared differences between the theoretical and experimental strains of the first composite beam; $\underline{x}_1 = [E_1^{(1)}, E_2^{(1)}, G_{12}^{(1)}, v_{12}^{(1)}]$, the vector containing the material constants considered at the first level with $x_{11} = E_1^{(1)}$, $x_{12} = E_2^{(1)}$, $x_{13} = G_{12}^{(1)}$, and $x_{14} = v_{12}^{(1)}$; ε_{x1}^* and ε_{y1}^* are the measured axial and lateral strains of the first composite beam, respectively. x_{1i}^L, x_{1i}^U are the lower and
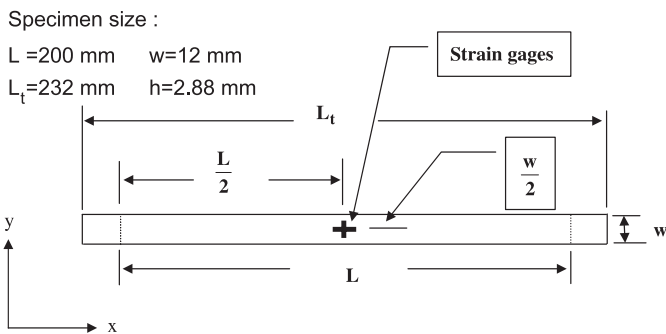
Specimen size :

L = 200 mm   w = 12 mm
L_t = 232 mm   h = 2.88 mm

Fig. 1. Dimensions of composite beam specimen.

upper bounds of the material constants, respectively; ε_{x1}, ε_{y1} are the theoretical strains determined in the strain analysis of the first composite beam using the trial values of the material constants; ξ is an amplification factor which is used to increase the sensitivity of and avoid the occurrence of numerical under-flow of the strain discrepancy function. A detailed numerical study has shown that for strains of graphite/epoxy or glass/epoxy symmetric angle-ply beams in the range from 10^{-3} to 10^{-5}, the value of ξ is best chosen in the range from 10^5 to 10^7. The above constrained minimization problem of Eq. (9) is first converted into an unconstrained minimization problem by creating the following general augmented Lagrangian [15]

$$\bar{\Psi}_1(\underline{\tilde{x}}_1,\underline{\mu},\underline{\eta},r_p) = e(\underline{\tilde{x}}_1) + \sum_{j=1}^{4}[\mu_j z_j + r_p z_j^2 + \eta_j \phi_j + r_p \phi_j^2] \tag{10}$$

with

$$z_j = \max\left[g_j(\tilde{x}_{1j}), \frac{-\mu_j}{2r_p}\right],$$
$$g_j(\tilde{x}_j) = \tilde{x}_{1j} - \tilde{x}_{1j}^{U} \leqslant 0,$$
$$\phi_j = \max\left[H_j(\tilde{x}_{1j}), \frac{-\eta_j}{2r_p}\right],$$
$$H_j(\tilde{x}_j) = \tilde{x}_{1j}^{L} - \tilde{x}_{1j} \leqslant 0; \quad j = 1-4, \tag{11}$$

where μ_j, η_j, r_p are multipliers; max [*,*] takes on the maximum value of the numbers in the bracket. The modified design variables $\underline{\tilde{x}}_1$ are defined as

$$\underline{\tilde{x}}_1 = \left[\frac{E_1^{(1)}}{\alpha_1}, \frac{E_2^{(1)}}{\alpha_2}, \frac{G_{12}^{(1)}}{\alpha_3}, \frac{v_{12}^{(1)}}{\alpha_4}\right], \tag{12}$$

where α_i are normalization factors used to adjust the magnitudes of the design variables. It is worth noting that the large differences among the values of the four material constants make the gradients of the strain discrepancy function with respect to E_1, E_2, and G_{12} much larger than that with respect to v_{12}. Therefore, in searching for the solution, the search direction will be so significantly dominated by the gradients of the strain discrepancy function with respect to E_1, E_2, and G_{12} that the solution may have great difficulty to converge. Herein, the normalization factors α_i are used to prevent E_1, E_2, and G_{12} from dominating the search direction of the solution and at the same time make the modified design variables have appropriate contributions to the search direction. A sensitivity study has shown that the solution of the above minimization problem can have excellent convergence rate if the normalization factors are chosen in such a way that they make the modified design variables less than 10 and greater than 0. It is noted that the modified design variables $\underline{\tilde{x}}_1$ are only used in the minimization algorithm while the original design variables $\underline{x}_1$ are used in the strain analysis of the composite

beam. The updated formulas for the multipliers μ_j, η_j, and r_p are

$$\mu_j^{n+1} = \mu_j^n + 2r_p^n z_j^n,$$
$$\eta_j^{n+1} = \eta_j^n + 2r_p^n \phi_j^n, \quad j = 1-4,$$
$$r_p^{n+1} = \begin{cases} \gamma_0 r_p^n & \text{if } r_p^{n+1} < r_p^{\max}, \\ r_p^{\max} & \text{if } r_p^{n+1} \geqslant r_p^{\max}, \end{cases} \tag{13}$$

where the superscript n denotes iteration number; γ_0 is a constant; $r_p^{\max}$ is the maximum value of r_p. Following the guideline given in the literature [15], the parameters μ_j^0, η_j^0, r_p^0, γ_0 and $r_p^{\max}$ are chosen as

$$\mu_j^0 = 1.0, \quad \eta_j^0 = 1.0, \quad j = 1-4,$$
$$\gamma_0 = 2.5, \quad r_p^{\max} = 100, \quad r_p^0 = 0.4. \tag{14}$$

The constrained minimization problem of Eq. (9) has thus become the solution of the following unconstrained optimization problem:

$$\text{minimize} \quad \bar{\Psi}(\underline{\tilde{x}}_1,\underline{\mu},\underline{\eta},r_p). \tag{15}$$

The solution of the above unconstrained optimization problem is straightforward by using the previously proposed unconstrained multi-start stochastic global minimization algorithm [16]. In the minimization process, a series of starting points for the modified design variables of Eq. (12) are selected at random from the region of interest. The lowest local minimum along the search trajectory initiated from each starting point is determined and recorded. A Bayesian argument is then used to establish the probability of the current overall minimum value of the objective function being the global minimum, given the number of starts and the number of times this value has been achieved. The multi-start optimization procedure is terminated once the condition that a target probability, typically 0.998, has been exceeded is satisfied. The estimates of the material constants G_{12} and v_{12} determined at this level of minimization are treated as the true values and thus kept constant in the second-level minimization problem which is expressed as

$$\text{minimize} \quad e(\underline{x}_2) = [(\varepsilon_{x2}^* - \varepsilon_{x2})^2 + (\varepsilon_{y2}^* - \varepsilon_{y2})^2] \cdot \xi$$
$$\text{subject to} \quad x_{2i}^L \leqslant x_{2i} \leqslant x_{2i}^U; \quad i = 1-2, \tag{16}$$

where $\underline{x}_2 = [E_1^{(2)}, E_2^{(2)}]$ the vector containing the material constants considered at the second level with $x_{21} = E_1^{(2)}$ and $x_{22} = E_2^{(2)}$; ε_{x2}^* and ε_{y2}^* are the measured axial and lateral strains of the second composite beam, respectively; ε_{x2}, ε_{y2} are, respectively, the theoretical axial and lateral strains determined in the strain analysis of the second composite beam using the trial values of E_1 and E_2. Again the above second-level minimization problem is solved using the same optimization technique as described in the first-level minimization problem.

4. Experimental investigation

A number of graphite/epoxy symmetric angle-ply beams with layups $[(45°/−45°)_6]_s$ and $[(60°/−60°)_6]_s$ and the dimensions shown in Fig. 1 were fabricated for the experimental study of the material constants identification of laminated composite materials. The material constants of the graphite/epoxy lamina were first determined using three types of the standard specimens in accordance with the ASTM standards of D3039 and D3518 and their average values and coefficients of variation (C.O.V.s) are given as follows:

$$E_1 = 146.5 \, \text{GPa} \ (0.7\%),$$
$$E_2 = 9.22 \, \text{GPa} \ (1.2\%),$$
$$G_{12} = 6.84 \, \text{GPa} \ (3.2\%),$$
$$v_{12} = 0.3 \ (0.19\%). \qquad (17)$$

The values in the above parentheses denote C.O.V.s. The composite beams comprised 24 laminae in which each lamina was 0.12 mm thick. The strains of the composite beams with different fiber angles predicted in the strain analyses of the beams using the actual values of the material constants in Eq. (17) are listed in Table 1 for future comparison. The $[(45°/−45°)_6]_s$ and $[(60°/−60°)_6]_s$ composite beams were subjected to three-point-bending tests in which two strain gages were used to measure the axial and lateral strains on the bottom surface at the mid-span of each of the composite beams. The strain gages produced by KYOWA, Japan had 3 mm gage length and

2.07 ± 1.0 gage factor. In the three-point-bending testing of the beams, the loading speed of the blade-like load applicator which had line contact with each beam was set as 0.02 mm/s. Three specimens of each beam type were tested and the load–strain relations of the specimens were constructed to produce the strain statistics for the identification of material constants. The measured strains together with their average values and C.O.V.s of the composite beams subjected to $F = 3$ N are listed in Table 2. It is noted that the C.O.V.s of the measured strains are less than or equal to 1.5%. The differences between the actual and the experimentally determined strains are less than or equal to 5.4%. The experimental strains will then be used in the present method to identify the material constants of the composite beams.

5. Sensitivity analysis

In practice, the existence of measurement noise is inevitable and such noise usually makes the measured strains behave like random variables of which the effects on the identified elastic constants can be studied via a probabilistic approach. Herein, an approximate analysis in the field of probability [17] is used to investigate the effects of the variations in measured strains on the accuracy of the identified elastic constants. To be conservative, the strains assumed to be measured independently can be treated as independent random variables. Let $(\overline{Y}_i, \sigma_i)$ be the expected value and standard deviation pair of measured strain Y_i. The elastic constants identified at each level of optimization can then be expressed as

$$X_i = G_i(Y_1, Y_2), \quad i = 1, 2. \qquad (18)$$

The expansion of X_i at the mean values of the measured strains in a truncated Taylor series gives

$$X_i \cong G_i(\overline{Y}) + \sum_{k=1}^{n} (Y_k - \overline{Y}_k) \frac{\partial G_i}{\partial Y_k}\bigg|_{\overline{Y}}. \qquad (19)$$

It is noted that the gradients $\frac{\partial G_i}{\partial Y_k}\big|_{\overline{Y}}$ are evaluated at the mean values of the measured strains. The determination of the gradients can be accomplished using the perturbation technique in which the differential changes of the elastic

Table 1
Actual strains of the graphite/epoxy $[(\theta°/−\theta°)_6]_s$ beams subjected to $F = 3$ N

Fiber angle θ	Strain	
	$\varepsilon_x \ (10^{-4})$	$\varepsilon_y \ (10^{-4})$
$[(15°/−15°)_6]_s$	0.7577	−0.7303
$[(30°/−30°)_6]_s$	1.597	−1.999
$[(45°/−45°)_6]_s$	3.869	−2.740
$[(60°/−60°)_6]_s$	6.922	−1.999

Table 2
Statistics of measured strains of the graphite/epoxy $[(\theta°/−\theta°)_6]_s$ beam subject to $F = 3$ N

Fiber angle θ	$\varepsilon_x^* \ (10^{-4})$				$\varepsilon_y^* \ (10^{-4})$			
	Specimen no.	Measured	Average	C.O.V.	Specimen no.	Measured	Average	C.O.V.
$[(45°/−45°)_6]_s$	1	3.879 (+0.3%)[a]	3.880 (+0.3%)	0.9%	1	−2.819 (+2.9%)	−2.858 (+4.3%)	1.2%
	2	3.844 (−0.6%)			2	−2.868 (+4.7%)		
	3	3.916 (+1.2%)			3	−2.887 (+5.4%)		
$[(60°/−60°)_6]_s$	1	7.045 (+1.8%)	7.081 (+2.3%)	1.3%	1	−2.023 (+1.2%)	−2.048 (+2.5%)	1.5%
	2	7.188 (+3.8%)			2	−2.081 (+4.1%)		
	3	7.009 (+1.3%)			3	−2.039 (+2.0%)		

[a]Value in parentheses denotes the percentage difference between actual and measured strains.

constants induced by the differential change of each measured strain are determined using the present identification method, and the finite difference method is used to calculate the gradients of the elastic constants with respect to the measured strain having the imposed differential change. For instance, the new set of measured strains $(Y_1 + \Delta Y_1, Y_2)$ is used to identify the elastic constants $X_i + \Delta X_i$ using the present elastic constants identification method. The gradients $\frac{\partial G_i}{\partial Y_1}|_{\overline{Y}}$ are then approximated as $\frac{\Delta X_i}{\Delta Y_1}$ via the finite difference approach. The first-order approximation to the variance of each elastic constant is

$$\text{var}[X_i] \cong \sum_{k=1}^{2} \left(\frac{\partial G_i}{\partial Y_k}\bigg|_{\overline{Y}} \right)^2 \text{var}[Y_k], \quad i = 1, 2, \tag{20}$$

where var[·] is the variance of the random variable in the brackets.

6. Results and discussion

The aforementioned two-level optimization method will be applied to the material characterization of laminated composite materials using two symmetric angle-ply beams. The upper and lower bounds of the material constants for different composite materials are chosen based on experience.

Graphite/epoxy:

$$0 < E_1 < 1000\,\text{GPa}, \quad 0 < E_2 < 50\,\text{GPa},$$
$$0 < G_{12} < 20\,\text{GPa}, \quad 0.1 < v_{12} < 0.5; \tag{21}$$

glass/epoxy:

$$0 < E_1 < 300\,\text{GPa}, \quad 0 < E_2 < 30\,\text{GPa},$$
$$0 < G_{12} < 20\,\text{GPa}, \quad 0.1 < v_{12} < 0.5.$$

The modified design variables of Eq. (12) are obtained via the use of the following normalization factors:

$$\alpha_1 = 1000, \quad \alpha_2 = 100, \quad \alpha_3 = 10, \quad \alpha_4 = 1. \tag{22}$$

The values of the amplification factor ξ in Eqs. (9) and (16) are set to be 10^6. It is noted that the use of the above values for the normalization and amplification factors can help increase the convergence rate of the solution. A number of numerical examples on the material constants identifications of graphite/epoxy and glass/epoxy composite laminates are first given to study the accuracy and feasibility of the proposed method. In the numerical study of the material characterization of graphite/epoxy composite laminates, the actual strains in Table 1 are treated as the "measured" strains for identifying the actual material constants given in Eq. (17). The "measured" strains of the $[(45°/-45°)_6]_s$ beam in Table 1 are used as an example to show the process of identifying the material constants in the first-level optimization problem. In this case, six starting points are randomly generated in getting the global minimum with probability exceeding 0.998. The numbers of iterations required for identifying the lowest local minima for the starting points and the material constants identified at the global minimum are tabulated in Table 3. It is noted that the exact values of G_{12} and v_{12} can be obtained for all the starting points with numbers of iterations less than or equal to 21 in the solution of the first-level optimization problem. This implies that the use of only one randomly generated starting point is enough to identify G_{12} and v_{12} for the $[(45°/-45°)_6]_s$ beam. It is also worth studying the results which will be obtained when the angle-ply beam with other fiber angle is used in the first-level optimization problem. Table 4 lists the identified material constants and their associated errors for the

Table 3
Identified material constants of the graphite/epoxy $[(45°/-45°)_6]_s$ beam at the first-level optimization

Starting point no.	Stage	Material constant				No. of iterations
		E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	v_{12}	
1	Initial	435.43	10.71	11.24	0.43	9
	Final	129.70	18.75	6.84	0.30	
2	Initial	306.25	4.27	17.98	0.26	20
	Final	129.70	18.75	6.84	0.30	
3	Initial	74.50	4.47	12.75	0.39	18
	Final	129.70	18.75	6.84	0.30	
4	Initial	523.17	41.81	15.66	0.24	19
	Final	129.70	18.75	6.84	0.30	
5	Initial	837.38	45.83	9.36	0.45	10
	Final	129.70	18.75	6.84	0.30	
6	Initial	685.09	14.82	8.54	0.35	21
	Final	129.70	18.75	6.84	0.30	
Global minimum		129.70 (11.5%)[a]	18.75 (103.4%)	6.84 (0%)	0.30 (0%)	Probability 0.999417

[a]Value in parentheses denotes percentage difference between identified and actual data.

Table 4
Identified material constants using the actual strains of the graphite/epoxy $[(\theta°/-\theta°)_6]_s$ beams at the first-level optimization

Fiber angle θ	Identified material constant			
	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	v_{12}
15°	151.65 (3.5%)[a]	14.59 (58.2%)	1.94 (71.6%)	0.32 (6.7%)
30°	185.32 (26.5%)	31.69 (243.7%)	3.12 (54.4%)	0.31 (3.3%)
45°	129.70 (11.5%)	18.75 (103.4%)	6.84 (0%)	0.30 (0%)
60°	No global minimum			

[a]Value in parentheses denotes percentage difference between identified and actual data.

Table 5
Data for material characterization of glass/epoxy angle-ply beams subject to $F = 1$ N

Material constants[a]	Beam layup	Actual strain	
		ε_x (10^{-5})	ε_y (10^{-5})
$E_1 = 38.6$ GPa	$[(15°/-15°)_6]_s$	8.957	−3.904
$E_2 = 8.27$ GPa	$[(30°/-30°)_6]_s$	13.86	−8.908
$G_{12} = 4.14$ GPa	$[(45°/-45°)_6]_s$	24.04	−12.36
$v_{12} = 0.26$	$[(60°/-60°)_6]_s$	32.96	−8.908

[a]Data from Swanson [14].

angle-ply beams under consideration. It is noted that except the $[(45°/-45°)_6]_s$ composite beams none of the adopted symmetrically laminated composite beams are able to identify the exact values of G_{12} and v_{12}. At the second-level optimization, the estimates of G_{12} and v_{12} identified in the previous level of optimization using the $[(45°/-45°)_6]_s$ beam are kept constant during the minimization process and the material constants E_1 and E_2 are identified using the "measured strains" of the angle-ply beam with fiber angle different from 45°. In solving the second-level optimization problem, the multi-start global minimization procedure is again used to randomly generate a number of starting points for the chosen angle-ply beam, search for the lowest local minima from the starting points, and identify the global minimum with probability greater than or equal to 0.998. It has been shown that all the adopted symmetric angle-ply beams can produce the exact estimates of E_1 and E_2 in an effective and efficient way. Next, consider the material characterization of glass/epoxy composite beams. The material constants, layups, and actual strains of the glass/epoxy beams used in the numerical study are listed in Table 5. It has been shown that amongst the adopted angle-ply beams, only the $[(45°/-45°)_6]_s$ beam can produce the exact values of G_{12} and v_{12} at the first-level optimization. On the other hand, all the symmetric angle-ply beams with fiber angles different from 45° under consideration can produce the exact values of E_1

and E_2 at the second-level optimization. It is thus obvious that the present two-level optimization method is capable to produce excellent identification of the material constants for different composite materials if the measured strains of the $[(45°/-45°)_6]_s$ beam and another symmetric angle-ply beam with fiber angle different from 45° are used to solve the first- and second-level optimization problems, respectively. For illustration purpose, the optimization algorithm DBCONF of IMSL [18] has also been used to solve the one-level optimization problem of Eq. (8) as well as the two-level minimization problem of Eqs. (9) and (16). It has been shown that for the cases which have been studied before, the optimization algorithm DBCONF is unable to make the solutions converge and thus no results are obtained.

The present method is now used to identify the material constants of the symmetric angle-ply beams which have been tested. The measured strains of each of the $[(45°/-45°)_6]_s$ beams as well as their average values in Table 2 are used separately at the first-level optimization to identify G_{12} and v_{12}. The identified estimates of G_{12} and v_{12} using different sets of measured strains are listed in Table 6. It is noted that excellent estimates of G_{12} and v_{12} with percentage differences less than or equal to 2.8%. In particular, the percentage differences of the estimates of G_{12} and v_{12} obtained using the average measured strains are 1.9% and 1%, respectively. At the second-level optimization, the values of G_{12} and v_{12} are set as 6.71 GPa and 0.297, respectively, which have been determined from the $[(45°/-45°)_6]_s$ beam at the previous level using the average measured strains while E_1 and E_2 are identified using different sets of measured strains of the $[(60°/-60°)_6]_s$ beams given in Table 2. The identified estimates of E_1 and E_2 at this level are listed in Table 7. Again it is noted that all the adopted sets of measured strains can produce excellent estimates of E_1 and E_2 with percentage differences less than 7.4%. In particular, the percentage differences of the estimates of E_1 and E_2 obtained using the average measured strains are 0.1%

Table 6
Identified material constants using measured strains of the graphite/epoxy $[(45°/-45°)_6]_s$ beams at the first-level optimization

Specimen no.	Identified material constant			
	E_1 (GPa)	E_2 (GPa)	G_{12} (GPa)	v_{12}
1	139.96 (4.5%)[a]	18.93 (105.3%)	6.75 (1.3%)	0.296 (1.3%)
2	154.41 (5.4%)	19.19 (108.1%)	6.74 (1.5%)	0.297 (1%)
3	145.02 (1.0%)	19.02 (106.3%)	6.65 (2.8%)	0.297 (1%)
Average strain	146.21 (0.2%)	19.04 (106.5%)	6.71 (1.9%)	0.297 (1%)
C.O.V.	5%	0.7%	0.8%	0.2%

[a]Value in parentheses denotes percentage difference between identified and actual data.

Table 7
Identified E_1 and E_2 using different measured strains of the graphite/epoxy $[(60°/-60°)_6]_s$ beams at the second-level optimization

Specimen no.	Identified elastic constant	
	E_1 (GPa)	E_2 (GPa)
1	137.44 (6.2%)[a]	9.10 (1.3%)
2	153.47 (4.8%)	8.54 (7.4%)
3	149.25 (1.9%)	9.21 (0.1%)
Average strain	146.71 (0.1%)	8.95 (2.9%)
C.O.V.	5.7%	4%

[a]Value in parentheses denotes percentage difference between identified and actual data.

and 2.9%, respectively. It is worth mentioning that the use of the other sets of the estimates of G_{12} and v_{12} in Table 6 at the second-level optimization can also produce excellent estimates of E_1 and E_2. Furthermore, the C.O.V.s of the identified elastic constants calculated using the sample data in Tables 6 and 7 are around 0.8%, 0.2%, 5.7%, and 4.0% for G_{12}, v_{12}, E_1, and E_2, respectively. The small C.O.V.s obtained for the identified elastic constants have thus further demonstrated the accuracy and repeatability of the present identification technique. For comparison purpose, two cases are given to further illustrate the merits of the present elastic constants identification method. In the first case, the one-level optimization problem of Eq. (8) is solved using the present multi-start global minimization technique together with the four average measured strains of the $[(45°/-45°)_6]_s$ and $[(60°/-60°)_6]_s$ beams given in Table 2. The solution of the one-level minimization problem produces the percentage differences of 13.5%, 2.2%, 1.9%, and 35% for E_1, E_2, G_{12}, and v_{12}, respectively. The relatively high percentage differences produced in the estimations of E_1 and v_{12} have demonstrated the fact that the elastic constants identified from the one-level minimization problem of Eq. (8) are very sensitive to measurement noise. The second case has been considered in Ref. [13] in which the axial, lateral, and shear strains measured from the graphite/epoxy $[(45°/-45°)_6]_s$ beam have been used to identify the four elastic constants of the graphite/epoxy composite material. The percentage differences of the estimates of E_1, E_2, G_{12}, and v_{12} obtained in the previous study were 6.3%, 3.9%, 3.5%, and 0%, respectively. When comparing with the identified elastic constants in Tables 6 and 7, it is noted that the percentage differences for the estimates of E_1, E_2, and G_{12} produced by the present method are lower than those produced by the previous method. Especially for the estimate of E_1, the percentage differences obtained by the present and previous methods are 0.1% and 6.3%, respectively. Therefore, it has further demonstrated that the present two-level optimization procedure has better capability to produce accurate estimates of elastic constants for composite materials than the aforementioned identification methods.

Finally, the effects of uncertainties encountered in strain measurements on the variations of the identified elastic constants are studied using the aforementioned sensitivity analysis. The C.O.V.s of the measured strains are assumed to be 2% in the sensitivity analysis of the elastic constants for the graphite/epoxy beams given in Table 1. It has been shown that at the first level of optimization, the theoretical C.O.V.s of the identified G_{12} and v_{12} obtained using the $[(45°/-45°)_6]_s$ beam are 1.46% and 0.24%, respectively. Such small effects on the variations of the identified G_{12} and v_{12} induced by the strain variations have also been observed in Table 6 where the experimental C.O.V.s of the identified G_{12} and v_{12} are 0.8% and 0.2%, respectively. At the second level of optimization, it has been shown that different symmetric angle-ply beams with fiber angles different from 45° produce different theoretical C.O.V.s for the identified E_1 and E_2. Among the symmetric angle-ply beams under consideration, the $[(15°/-15°)_6]_s$ beam can produce the smallest C.O.V.s of 2.2% and 3.9% for the identified E_1 and E_2, respectively. Therefore, it is recommended that the $[(45°/-45°)_6]_s$ and $[(15°/-15°)_6]_s$ beams be used at the first and second levels of optimization, respectively, in the present procedure to identify the elastic constants of laminated composite beams.

7. Conclusions

A two-level optimization procedure for the identification of four material constants of fiber-reinforced composite materials using four strains measured from two symmetric angle-ply beams with different fiber angles subjected to three-point-bending testing has been presented. In the proposed method, a $[(45°/-45°)_6]_s$ beam and a symmetric angle-ply beam with fiber angle different from 45° have been used, respectively, at the first-level optimization to identify G_{12} and v_{12} and at the second-level optimization to identify E_1 and E_2. A number of numerical examples on the material characterization of graphite/epoxy and glass/epoxy beams have been given to demonstrate the capability and accuracy of the present method in identifying the material constants of laminated composite materials. The theoretical study has shown that the present method can produce exact estimates of the material constants for the beams made of different composite materials in an efficient and effective way. Static three-point-bending tests of several $[(45°/-45°)_6]_s$ and $[(60°/-60°)_6]_s$ composite beams have been performed to measure the axial and lateral strains of the composite beams, along with the experimental data used to study the feasibility and accuracy of the present method. The experimental study has also shown that the use of $[(45°/-45°)_6]_s$ and $[(60°/-60°)_6]_s$ in the first and second levels of optimization problems of the present method can produce good estimates of the elastic constants for the laminated composite beams in an effective and efficient way. The percentage differences in the identification of material constants G_{12}, v_{12}, E_1, and E_2 are less than or equal to 2.9% for the case where the

average measured strains of the $[(45°/−45°)_6]_s$ and $[(60°/−60°)_6]_s$ composite beams are used at the first and second levels of optimization, respectively. A sensitivity analysis has been given to show that the use of a $[(15°/−15°)_6]_s$ beam in solving the second-level optimization problem of the present procedure can identify E_1 and E_2 with less variations. The present method has the potential to become a useful tool for the determination of material constants.

Acknowledgment

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